# The instability associated with the onset of motion in a thermosyphon

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Abstract—A theoretical method is presented for the study of the onset of motion in a symmetrical natural circulation loop. A one-dimensional model is applied to describe the behavior of the flow and linear stability analysis is used for the investigation of its stability. The results show that there exists a critical modified Rayleigh number,  $R_c$ , below which the rest state is stable and any flow perturbation will decay. The same results have been obtained from a steady-state analysis; there is no steady-state solution when  $R < R_c$ . For a vertical loop composed of two parallel branches heated from below and cooled from above,  $R_c = 6$ .

#### 1. INTRODUCTION

FREE CONVECTION loops (thermosyphons), created by heating from below and cooling from above, have applications in geophysical processes and energy conversion systems. It is well known that natural circulation flows may be unstable, cf. refs. [1–20]. There are four different types of instabilities in such loops, which have been found theoretically and experimentally [1]:

- (a) The onset of local closed cells of the Rayleigh-Bénard type, when a certain critical Rayleigh number is exceeded, similar to other thermal instabilities (this point is further discussed below).
- (b) The onset of global flow around the loop, when further heating is supplied [3, 9, 10, 13].
- (c) The instability of steady flow in the loop, associated with oscillation growth [3-13, 16].
- (d) Multiple steady-state solutions, indicating metastable instabilities [2, 11, 12, 14, 15].

The stability characteristics of a thermosyphon have important implications. On one hand, the flow is created by a thermal instability. Thus in an energy conversion system based on natural circulation, favorable conditions for the instability of the second kind mentioned above must be ascertained. In such systems it is desired on the other hand, to remove the energy from the heat source in a stable flow. Therefore instabilities of the third and fourth kind should be avoided.

The present paper deals with the instability of the second kind, i.e. associated with the onset of global flow around the loop. The treatment here bears also a relation to other situations, such as the onset of flow in stagnant branches in a multi-channel natural circulation system, cf. [13].

Evidently, any asymmetric loop, where heating is applied (even partly) from the side, is unstable as regards the onset of motion. This case is usually encountered in energy conversion systems, i.e. thermosyphonic solar water heaters [5] and emergency

core cooling of nuclear reactors [1, 13-15]. The stability of the second kind in symmetric loops is an interesting problem. Welander [3] stated that the rest state of a loop with two vertical branches, a point heat source at the bottom and a point heat sink at the top, is always unstable. The same conclusion has also been reached by Bau and Torrance [10], who investigated an open symmetric free convection loop. However, in the latter analysis a linear distribution of the temperature perturbation was assumed. As has been shown by Zvirin and Greif [6], this assumption is not always justified: it leads to the result that the flow in the vertical loop treated by Welander [3] is stable even in cases where the exact solution leads to instabilities. Torrance and Chan [9] obtained a critical Rayleigh number for the onset of motion in another open thermosyphon. This result was achieved by an asymptotic expansion of the condition for a steady flow in the loop, without the explicit heat conduction term in the energy equation. These terms are usually neglected in the steady-state solutions, because they are small compared to the convection terms.

Yorke and Yorke [18], Hart [19] and Malkus [20] investigated instabilities in a toroidal thermosyphon. They have found that the no-flow solution is unstable when a modified Rayleigh number exceeds a certain critical value. However, the analyses of [18-20] again failed to include the conduction term in the energy equation. Moreover, some rather restricting assumptions have been made concerning symmetry conditions of the driving forces (heat fluxes and wall temperatures). For example, the results do not apply to the case of heating by a uniform heat flux from below and cooling by a constant wall temperature at the upper half, treated experimentally and theoretically in refs. [4] and [7]. Finally, the energy equation for the steady rest state in a channel heated by a prescribed flux cannot be satisfied without the heat conduction term. As has also been indicated by [10], axial conduction is the only mechanism which enables heat transfer from the heat source to the sink.

In this work a symmetric vertical thermosyphon is

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### NOMENCLATURE

A cross-sectional area

a radius of pipe

 $d_{\rm H}$  hydraulic diameter of the flow channel

f friction factor

q acceleration of gravity

h height of the loop

P modified Prandtl number, equation (10)

Q volumetric flowrate

R modified Rayleigh number, equation (10)

Re Reynolds number

s coordinate around the loop

T temperature

t time

V velocity

 $V_{\rm ch}$  characteristic velocity, equation (6b)

v dimensionless velocity

w dimensionless velocity, equation (12)

z vertical coordinate.

Greek symbols

α thermal diffusivity

β thermal expansion coefficient

 $\theta$  dimensionless temperature

 $\lambda$  parameter, equation (24)

v kinematic viscosity

 $\rho$  density

 $\sigma$  stability parameter

 $\phi$  velocity perturbation

 $\psi$  temperature perturbation.

Subscripts

c critical (Rayleigh number)

D lower container

l left branch

r right branch

U upper container.

Superscript

steady state.

considered. The stability margin of the rest state is sought, which corresponds to the onset of a global motion around the loop. A one-dimensional model is adopted, because we are interested in the initial state of a disturbance in the form of a flow around the whole loop, which will grow to establish the global flow. The heat conduction terms in the energy equations are maintained and the method of linear stability analysis is utilized.

The main result of the stability study is that there exists a critical modified Rayleigh number,  $R_c$ , for the onset of motion. When  $R > R_c$  a steady flow will be established, while for  $R < R_c$  any flow disturbance will decay and the rest state will be resumed.

## 2. THE GOVERNING EQUATIONS

Consider the loop shown schematically in Fig. 1. It consists of two vertical insulated branches connecting two rigid containers. The fluid completely fills the system and its temperature is maintained constant,  $T_{\rm D}$  and  $T_{\rm U}$  in the lower and upper containers. Heating from below is investigated, i.e.  $T_{\rm D} > T_{\rm U}$ .

We are interested here in the onset of global motion around the loop from a rest state. Thus the velocities developed (in the initial stages) are small and laminar flow prevails. The Boussinesq approximation is adopted, whereby the density,  $\rho$ , is taken as constant in the governing equations except for the body force term in the momentum equation, where  $\rho = \rho_o [1 - \beta(T - T_o)]$ .  $\rho_o$  is the density at the reference temperature,  $T_o$ , chosen for convenience as  $T_o = T_U$ .

A one-dimensional model is used, with V and T

representing cross-sectional average velocity and temperature in the vertical branches. The single spatial coordinate, s, runs around the loop.

Continuity considerations imply that any flow in a branch (say, upwards in the right branch) is coupled to a flow in the other one with the same flow rate, Q, and in the opposite direction. Furthermore, the one-dimensional continuity equation yields the result that the velocity is a function of the time only:

$$V = V(t). (1)$$

The momentum equation is integrated around the loop, to eliminate the pressure term. It is written in the following general form, cf. [1]:

$$\rho_{o}\left(\oint \frac{\mathrm{d}s}{A}\right) \frac{\mathrm{d}Q}{\mathrm{d}t} = -g \oint \rho \,\mathrm{d}z - 2\rho_{o} \oint f V^{2} \frac{\mathrm{d}s}{\mathrm{d}_{H}} \quad (2)$$

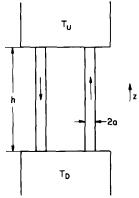


Fig. 1. A scheme of the vertical thermosyphon.

where z is the vertical coordinate, A is the cross-sectional area, V is the velocity (V = Q/A), f is the friction factor and  $d_H$  is the hydraulic diameter of the flow channel.

For simplicity we take the vertical branches to be circular pipes with radius a. The friction factor for laminar flow in these pipes is taken as f = 16/Re = 16v/2aV. It is assumed that the pipes are long enough so that the pressure drops in the connections to the containers are negligible and are wide enough so that the effect of surface tension is vanishingly small. Equation (2) then reduces to:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\beta g}{2h} \int_0^h (T_r - T_l) \,\mathrm{d}z - \frac{8\nu V}{a^2} \tag{3}$$

where h is the height of the loop (see Fig. 1) and the subscripts r and l denote the right and left branches.

The energy equation for the insulated pipes is:

$$\frac{\partial T}{\partial t} \pm V \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2}, \quad +r \\ -1. \tag{4}$$

Viscous dissipation has evidently been neglected in the equation. However, the heat conduction term, neglected in most analyses of thermosyphons compared to the convection terms, must be retained here, because it governs the steady initial temperature distribution. It is noted that Mertol [17] has solved numerically the equations for the toroidal loop, including the conduction terms, but he did not study the problem of the onset of motion.

Finally, the boundary conditions for the temperature are determined by the containers:

$$T_{\rm r} = T_{\rm l} = T_{\rm D} \quad \text{at } z = 0 \tag{5a}$$

$$T_{\rm r} = T_{\rm l} = T_{\rm U} \quad \text{at } z = h. \tag{5b}$$

The variables are transformed to dimensionless form by the following scaling:

length 
$$h$$
 (6a)

velocities\* 
$$V_{\rm ch} = \beta g (T_{\rm D} - T_{\rm U}) a^2 / 16 v$$
 (6b)

time 
$$h/V_{\rm ch}$$
 (6c)

temperature 
$$\theta = (T - T_U)/(T_D - T_U)$$
. (6d)

Introduction of (6) into (3)–(5) leads to the dimensionless governing equations:

$$\frac{\partial v}{\partial t} = \int_0^1 (\theta_r - \theta_l) \, \mathrm{d}z - v \tag{7}$$

$$P\frac{\partial\theta}{\partial t} \pm Rv\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial z^2}, + r$$
(8)

$$\theta_r = \theta_1 = 1 \quad \text{at } z = 0 \tag{9a}$$

$$\theta_{\rm r} = \theta_{\rm l} = 0 \quad \text{at } z = 1 \tag{9b}$$

where t and z now denote the non-dimensional variables,  $v \equiv V/V_{\rm ch}$  and the parameters P and R are modified Prandtl and Rayleigh numbers, defined by:

$$P = 8\left(\frac{h}{a}\right)^{2} \frac{v}{\alpha} = 8\left(\frac{h}{a}\right)^{2} Pr;$$

$$R = \frac{V_{ch}h}{\alpha} = \frac{\beta g(T_{D} - T_{U})a^{2}h}{16\alpha v}.$$
(10)

#### 3. THE STEADY-STATE SOLUTION

We seek, first, the solution of equations (7)–(9) for the steady state, i.e. the time derivatives do not appear in the equations. The steady-state velocity, v, is constant around the loop. Equations (8) can therefore be solved for  $\theta(z)$  with v as a parameter, which is as yet unknown.

The solution for the steady temperature distributions in the right and left branches of the loop is obtained from the equations (8) and the boundary conditions (9) as:

$$\theta_{\rm r} = \frac{{\rm e}^{wz} - {\rm e}^{w}}{1 - {\rm e}^{w}}; \quad \theta_{\rm l} = \frac{{\rm e}^{-wz} - {\rm e}^{-w}}{1 - {\rm e}^{-w}}$$
 (11)

where:

$$w \equiv Rv. \tag{12}$$

These distributions are introduced into the momentum equation (7) for the steady state. After the integrations are carried out an algebraic equation for the velocity v (or w) is obtained:

$$\frac{e^w + 1}{e^w - 1} - \frac{2}{w} = v = \frac{w}{R}.$$
 (13)

The solution of the last equation gives the steadystate velocity in the loop. It can be introduced into (11) in order to find the temperature distributions. The steady-state solution is seen to depend on a single parameter, namely the modified Rayleigh number, R.

The main interest here is the onset of motion, where the velocities are small. Therefore, a solution of equation (13) is sought, first, for  $w \ll 1$ . A careful asymptotic expansion of the equation† leads to the solution:

$$w = \left(\frac{R - 6}{1 - (3R/30)}\right)^{1/2}, \quad w \ll 1. \tag{14}$$

An important result immediately emerges: there is no steady flow in the loop if the Rayleigh number is smaller than a critical value. For the thermosyphon under consideration:

$$R_{\rm c} = 6. \tag{15}$$

The velocity, w, at values of R just above the critical [equation (15)] is obtained by inserting R = 6 in the

<sup>\*</sup> $V_{\rm ch}$  is the solution for the steady-state velocity for the case where the conduction term is neglected in the energy equations.

<sup>†</sup> In order to obtain the leading-term approximation (14), it is necessary to carry the expansions to fifth order.

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denominator of equation (14):

$$w = \sqrt{10(R-6)}$$
  $(R \approx 6, w \ll 1)$ . (16)

It is noted that equations (14) and (16) can be used only when  $R \approx 6$  and  $w \ll 1$ . At  $R \approx 6\frac{2}{3}$ , for example, (14) cannot be applied because w would be too large.

The other limiting case, for large values of w, is easily obtained from equation (13) as w = R, or, by equation (12), v = 1. This means that for large values of R,  $V = V_{\rm ch}$  and as mentioned above it is the solution for the limiting case where the conduction terms are small compared to the convection terms (a more exact expression for large R is w = R - 2).

For intermediate values of w, a numerical solution of equation (13) is needed for w(R). However, the solution procedure can be simplified by calculating R as a function of w from this equation (and also noting that the function is monotonously increasing). The steady-state solution is symmetric with respect to w, in that for any value w, -w is also a solution. This behavior can be observed from equations (13) and (14), and also from the governing equations. The physical meaning of this phenomenon is that the two branches of the loop are symmetric and do not have any preference.

The results for the steady dimensionless velocity w as a function of the modified Rayleigh number, R, are shown in Fig. 2. It can be seen that the approximation for small w is very good only in a narrow range of R ( $6 \le R < \sim 6.1$ ). The asymptotic expression for large R becomes accurate at  $R \approx 20$ . Finally, it is reminded that the non-dimensional velocity v is given by v = w/R, and the dimensional one by  $V = vV_{\rm ch}$ .

# 4. THE ONSET OF MOTION—STABILITY OF THE REST STATE

In the previous section it has been found that no steady flow can be established in the loop when the modified Rayleigh number, R, is below a critical value,  $R_c$ . We proceed now to investigate the instability associated with the onset of motion in the loop.

The rest state is governed by equations (7)–(9) without the time derivatives and with  $\bar{v} = 0$ . The (trivial) exact solution in this case is:

$$\bar{\theta}_{\rm r} = \bar{\theta}_{\rm l} = \bar{\theta} = 1 - z \tag{17}$$

which obviously satisfies the momentum equation (7), in that there is no net buoyancy force, due to symmetry.

Linearized stability analysis is used for the stability study. We therefore write the time-dependent velocity and temperature as the steady (rest) values, with small superimposed perturbations:

$$v = \phi e^{\sigma t}; \quad \theta = \overline{\theta} + \psi(s) e^{\sigma t}$$
 (18)

where  $\sigma$  is the stability parameter. In general,  $\sigma$  may be complex and the state is unstable when Real  $(\sigma) > 0$ .

Introduction of these expressions into the momentum and energy equations (7) and (8) yields:

$$\phi(\sigma+1) = \int_0^1 (\psi_r - \psi_l) dz$$
 (19)

$$\frac{\mathrm{d}^2 \psi_{\mathrm{r,l}}}{\mathrm{d}z^2} - P \sigma \psi_{\mathrm{r,l}} = \mp R \phi, \quad \frac{-\mathrm{r}}{+1} \tag{20}$$

with the boundary conditions:

$$\psi_{r,1} = 0$$
 at  $z = 0, 1$ . (21)

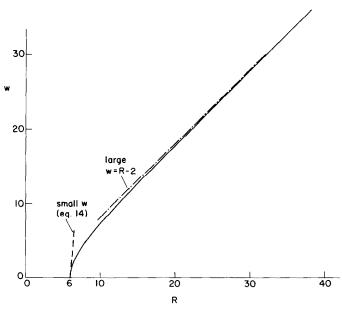


FIG. 2. The dimensionless steady-state velocity, w, as a function of the modified Rayleigh number, R.

It is readily seen that due to symmetry of the equations,  $\psi_1 = -\psi_r$ . It is therefore sufficient to solve for  $\psi_r = \psi$  only, and equation (19) reduces to:

$$\phi(\sigma+1) = 2\int_0^1 \psi \, dz.$$
 (22)

The solution procedure is similar to that of the steady-state problem (Section 3). The energy equation (20) with the boundary conditions (21) is first solved for  $\psi$ , in terms of  $\phi$  as a constant parameter (unknown at this stage):

$$\psi = \frac{R\phi}{P\sigma} \left( \frac{e^{-\lambda} - 1}{e^{\lambda} - e^{-\lambda}} e^{\lambda z} + \frac{1 - e^{\lambda}}{e^{\lambda} - e^{-\lambda}} e^{-\lambda z} + 1 \right) \quad (23)$$

where:

$$\lambda^2 \equiv P\sigma. \tag{24}$$

Introduction of (23) into (22) and integration yields the characteristic equation for the stability parameter  $\sigma$ :

$$\frac{2R}{P\sigma}\left(1 - \frac{2}{\lambda}\frac{e^{\lambda} - 1}{e^{\lambda} + 1}\right) - (\sigma + 1) = 0. \tag{25}$$

The primary interest is in the neighborhood of  $R = R_c = 6$ : it was shown in the previous section that there is no steady-state solution when  $R < R_c$  and a single one when  $R > R_c$ . By a careful expansion of equation (25) near R = 6 it is found that there exists, then, a double solution for  $\lambda$ , with  $|\lambda| \ll 1$ , given by:

$$\lambda = \pm \left[ \frac{2(R-6)}{3\left(\frac{4}{P} + 1 - \frac{R}{10}\right)} \right]^{1/2}, \quad 0 < R-6 \ll 1. \tag{26}$$

By introduction of R = 6 in the denominator of the last relationship we obtain:

$$\lambda = \pm \left[ \frac{R - 6}{3\left(\frac{2}{P} + \frac{1}{5}\right)} \right]^{1/2}, \quad 0 < R - 6 \ll 1. \quad (27)$$

It is now necessary to consider separately the cases R > 6 and R < 6 (P is always positive). For the former, there are two real roots for  $\lambda$ , and from equation (24) we find the following single value for the stability parameter,  $\sigma$ :

$$\sigma = \frac{R - 6}{3\left(2 + \frac{P}{5}\right)} > 0, \quad 0 < R - 6 \ll 1.$$
 (28)

For R < 6, equation (26) yields a double imaginary solution for  $\lambda$ , which leads to the following single solution for  $\sigma$ :

$$\sigma = -\frac{6 - R}{3\left(2 + \frac{P}{5}\right)} < 0, \quad 0 < 6 - R \ll 1. \tag{29}$$

The results (28) and (29) show that for all values of the modified Prandtl number P, the stability parameter,  $\sigma$ , has a single value near the critical modified Rayleigh

number  $R_c = 6$ . Thus,  $\sigma$  is positive and the rest state is unstable when  $R > R_c$  and  $\sigma$  is negative and the rest state is stable when  $R < R_c$ . This confirms the same result obtained in the previous section on the basis of the steady-state analysis.

For large values of the ratio R/P, equation (25) has the following double real root:

$$\sigma = \pm \sqrt{2R/P}, \quad R/P \gg 1. \tag{30}$$

A numerical investigation of the characteristic equation (25) reveals that it always has a real root for R > 6 and for any value of P. The magnitude of the root increases with R and decreases with P, as can be seen from Fig. 3. It is therefore concluded that when R > 6 the rest state of the thermosyphon is always unstable; there exists a monotonous mode of perturbation growth, with a rate which increases with R and decreases with P.

The result of equation (29) indicates that the rest state of the loop is stable for  $0 < 6 - R \ll 1$ . It is expected that the whole range R < 6 (at any value of P) is stable. In order to prove this statement, it is necessary to consider all arbitrary perturbation modes, including oscillatory ones. For this the Nyquist criterion has been used, cf. [4]. The results show that for R < 6 and all P, equation (25) does not have any root with a positive real part and all these states are therefore stable.

The onset of local closed cells of the Rayleigh-Bénard type is determined by the Rayleigh number  $Ra = \beta g \Delta T h^3 / v \alpha$ . Thus  $Ra/R = 16(h/a)^2$  and for example the value of Ra corresponding to  $R_c = 6$  and  $h/a \sim 10$  is of the order  $10^4$ . This indicates that such cells may form

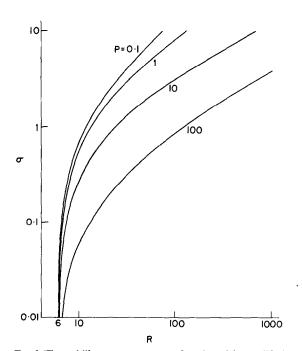


FIG. 3. The stability parameter,  $\sigma$ , as a function of the modified Rayleigh and Prandtl numbers, R and P.

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in certain cases before the onset of global motion around the loop, depending on the loop parameters.

### CONCLUSIONS

The stability associated with the onset of motion in a thermosyphon has been investigated. The main result is that there exists a critical value,  $R_c$ , of the modified Rayleigh number. For  $R > R_c$  the rest state of the loop is always unstable and there exists a monotonously growing perturbation, whose rate increases with R and decreases with the modified Prandtl number, P. When  $R < R_c$  the rest state is unconditionally stable. For the vertical thermosyphon treated here,  $R_c = 6$ . This result has been obtained in two different ways: by a stability analysis and by the solution of the steady-state problem. The latter shows that there is no flow solution for  $R < R_c$ .

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# INSTABILITE ASSOCIEE A L'APPARITION DU MOUVEMENT DANS UN THERMOSIPHON

**Résumé**—Une méthode théorique est présentée pour l'étude de l'apparition du mouvement dans une boucle symétrique de circulation naturelle. Un modèle monodimensionnel est appliqué pour décrire le comportement de l'écoulement et une analyse linéaire de stabilité est utilisée pour l'étude de sa stabilité. Les résultats montrent qu'il existe un nombre modifié de Rayleigh critique  $R_{\rm e}$ , au dessous duquel l'état de repos est stable et pour lequel une perturbation quelconque de l'écoulement s'atténue. Les mêmes résultats ont été obtenus à partir d'une analyse de régime permanent; il n'y a pas de solution du régime permanent quand  $R < R_{\rm e}$ . Pour une boucle verticale composée de deux branches parallèles, chauffée à la base et refroidie au sommet  $R_{\rm e} = 6$ .

# DIE INSTABILITÄT, DIE ZUM EINSETZEN DER STRÖMUNG IN EINEM THERMOSYPHON FÜHRT

Zusammenfassung—Eine theoretische Methode zur Untersuchung des Einsetzens der Strömung in einem symmetrischen Naturamlauf wird vorgestellt. Das Verhalten der Strömung wird mit Hilfe eines eindimensionalen Modells beschrieben, die Stabilität unter Anwendung einer linearen Stabilitäts-Analyse untersucht. Die Ergebnisse zeigen, daß eine modifizierte kritische Rayleigh-Zahl  $R_c$  existiert, unterhalb der ein stabiler Ruhezustand herrscht und jede Störung abklingt. Die gleichen Ergebnisse ergeben sich bei der stationären Analyse, wobei es keine stationäre Lösung gibt, wenn  $R < R_c$ . Bei vertikaler Anordnung, bestehend aus zwei parallelen unten beheizten und oben gekühlten Ästen, ist  $R_c = 6$ .

## НЕУСТОЙЧИВОСТЬ, СВЯЗАННАЯ С НАЧАЛОМ ДВИЖЕНИЯ В ТЕРМОСИФОНЕ

**Аннотация**—Представлен теоретический метод для исследования начала движения в контуре с симметричной естественной циркуляцией. Одномерная модель применяется для описания поведения течения, а при исследовании устойчивости используется линейный анализ. Результаты показывают, что существует критическое модифицированное число Рэлея  $R_{\rm c}$ , ниже которого состояние покоя является устойчивым и любые возмущения потока затухают. Такие же результаты получены из анализа стационарного состояния; при  $R < R_{\rm c}$  не существует стационарного решения. Для вертикального контура, состоящего из двух параллельных ответвлений, нагреваемых снизу и охлаждаемых сверху,  $R_{\rm c} = 6$ .